PHYS 2601 (Fall 2025): Problem Set 4
Due date: October 5, 9am. 50% penalty on late homework.

From Vibrations and Waves (King)
Problem 3.8 (5 pts)
Problem 3.12 (5 pts)
Problem 4.1 (5 pts)
Problem 4.5 (5 pts)

From Vibrations and Waves (French)
Problem 4.5 (10 pts)

4–5 A simple pendulum has a length (l) of 1 m. In free vibration the amplitude of its swings falls off by a factor e in 50 swings. The pendulum is set into forced vibration by moving its point of suspension horizontally in SHM with an amplitude of 1 mm.

(a) Show that if the horizontal displacement of the pendulum bob is x, and the horizontal displacement of the support is ξ, the equation of motion of the bob for small oscillations is

$$\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \frac{g}{l}x = \frac{g}{l}\xi$$

Solve this equation for steady-state motion, if $\xi = \xi_0 \cos \omega t$. (Put $\omega_0{}^2 = g/l$.)

(b) At exact resonance, what is the amplitude of the motion of the pendulum bob? (First, use the given information to find Q.)

(c) At what angular frequencies is the amplitude half of its resonant value?

(see extra credit next page)

Extra credit problem: Numerical solution of the forced damped simple pendulum (5 points)

In class, we have discussed forced oscillations. To find an analytic solution to the problem, we have made the assumption that the oscillator always oscillates at the frequency ω of the drive. In this problem, we will find that this is not the complete story. You may recall that in the experiment that we did in class, we saw a beating of oscillations, especially at the start of the experiment. By numerically solving the problem of forced oscillations, you will reveal the details of this phenomenon.

(a) As in Problem Set 3, we use the example of a simple pendulum. Numerically determine the displacement θ of the pendulum as a function of time by numerically solving the differential equation:

$$\ddot{\theta}(t) + \frac{g}{l}\sin\theta(t) + \frac{b}{m}\dot{\theta}(t) = \frac{F_0}{m}\cos\omega t. \tag{1}$$

To implement the numerical approach you can use a software of your choice, e.g., Excel, Python, or Mathematica. For the length of the pendulum use $l = g/\pi^2$, where g is the gravitational acceleration of the Earth. For the damping constant use $b = 0.15$, for the mass $m = 1$, and for the force amplitude $F_0 = 1$. **Provide a description and a screen shot of your code.**

(b) Determine the displacement as a function of time for the following parameters:

(i) $\theta(t=0) = 75°$, $\dot{\theta}(t=0) = 0$, $\delta t = 0.02$s, and $\omega = 0.1$

(ii) $\theta(t=0) = 75°$, $\dot{\theta}(t=0) = 0$, $\delta t = 0.02$s, and $\omega = 1.5$

(iii) $\theta(t=0) = 75°$, $\dot{\theta}(t=0) = 0$, $\delta t = 0.02$s, and $\omega = 3.14$

(iv) $\theta(t=0) = 75°$, $\dot{\theta}(t=0) = 0$, $\delta t = 0.02$s, and $\omega = 3.5$

(v) $\theta(t=0) = 75°$, $\dot{\theta}(t=0) = 0$, $\delta t = 0.02$s, and $\omega = 5$

(vi) $\theta(t=0) = 75°$, $\dot{\theta}(t=0) = 0$, $\delta t = 0.02$s, and $\omega = 10$

Plot the displacement as a function of time up to 20 s. Provide a table with the first few time steps listing the values of time t, $\theta(t)$, and $\dot{\theta}(t)$ for both cases.

(c) Compare the plots for the different ω values. How do you interpret your findings for low, medium, and high values of ω? Also, compare to the frequency $\sqrt{\omega_0 - \gamma^2/2}$ at which we expect the peak amplitude for a forced oscillator. Does the system reach the peak amplitude at this value? For some additional background information, read section 3.5 on *Transient Phenomena* in the book by King.

In your solutions, please provide written comments (in addition to the math) that show your reasoning to receive full credit.

Please submit solutions electronically as a pdf document to gradescope (handwritten or typeset).